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Spacelike Fluctuations of the Stress Tensor for de Sitter Vacuum

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The two-point function characterizing the stress tensor fluctuations of a massless, minimally coupled field for an invariant vacuum state in de Sitter spacetime is discussed. This two-point function is explicitly computed for spacelike-separated points which are geodesically connected. We show that these fluctuations are as important as the expectation value of the stress tensor itself. These quantum field fluctuations will induce fluctuations in the geometry of de Sitter spacetime. This paper is a first step toward the computation of such metric fluctuations, which may be of interest for large-scale structure formation in cosmology. The relevance of our results in this context is briefly discussed.

1. INTRODUCTION

In today's standard inflationary scenario the amplified vacuum fluctuations of the inflaton field provide the seeds for large-scale structure formation [1–3]. It is thus of interest to understand the effect of these quantum fluctuations on the universe dynamics and the implications they may have for cosmological observations.

In most inflationary models (exponential inflation), the geometry of spacetime during the period of inflation can be reasonably well described by de Sitter spacetime. This spacetime also seems favored by quantum cosmology since most accepted "initial condition" proposals, which are either quantum "tunneling from nothing" [4] or a "no-boundary" condition [5], both predict it.

It has been recently pointed out that fluctuations in the stress-energy tensor of quantum fields may be important for some states in curved spacetimes or even flat spacetimes with nontrivial topology [6]. Hu and Phillips, for instance, computed the energy density fluctuations of quantum states in spatially closed

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Friedmann–Robertson–Walker models and showed that these could be as important as the energy density itself [7]. If so, these fluctuations may induce relevant backreaction effects on the gravitational field (the spacetime geometry).

In this paper we compute the fluctuations of the stress-energy tensor for a scalar field in de Sitter spacetime. We consider a massless, minimally coupled field in the Euclidean vacuum state, which is a de Sitter-invariant state. The motivation for considering the massless minimal coupling case is the fact that it mimics the behavior of gravitons in a curved background as well as that of the perturbations of the inflaton field in usual inflationary models. As for the state, the reasons why we chose the Euclidean vacuum are twofold. On one hand, the high degree of symmetry makes the computations simpler. On the other hand, there are at least two serious physical motivations. First, it naturally arises in exponential inflation models, and for a massive field it has been shown to be the state to which any other state tends asymptotically in a de Sitter background [8]; second, it is selected by the most popular boundary conditions in quantum cosmological models [9].

Here we compute the two-point correlation for spacelike-separated points of the stress-energy tensor and find that these fluctuations are important. Therefore the backreaction of these fluctuations on the spacetime geometry could be relevant, and will be the subject of further work within the context of stochastic semiclassical gravity and the Einstein–Langevin equation [10]. Related to this fact, it is worth mentioning that Abramo *et al.* [11, 12] have shown that the second-order contribution to the backreaction of inflaton and gravitational perturbations during the inflationary period can be important even below the "self-reproduction" scale. Although the spirit of our approach to the backreaction problem is slightly different, we believe a partial connection with their work may exist. These issues will also be addressed in future investigations.

The plan of the paper is the following. In Section 2 we give a very brief review of de Sitter spacetime, its properties, and the solution of the Klein–Gordon equation in such a background. In Section 3, following the proposal made in ref. 13, we discuss the invariant vacuum state for the massless, minimally coupled field that we are going to use. In Section 4 we deal with the fluctuations of the stress-energy tensor of the scalar field in the de Sitter background and compute the noise kernel (i.e., the two-point correlations) which characterizes these fluctuations for spacelike-separated points. We discuss our results in the final section. Throughout the paper we will use the $(+, +, +)$ sign convention of Misner *et al.* [14].

2. DE SITTER SPACETIME

In this section we give a brief summary of some useful properties and definitions related to de Sitter spacetime. For a more detailed exposition, see refs. 15 and 16.

Four-dimensional de Sitter spacetime has positive constant curvature and is thus maximally symmetric. Its 10-dimensional group of isometries is $O(4, 1)$. It can be represented as a hyperboloid embedded in a five-dimensional spacetime with Minkowskian metric η_{AB}, so that $\eta_{AB}\xi^A(x)\xi^B(x) = H^{-2}$, where the Hubble constant H is related to the scalar curvature $R = 12H^2$ and $\xi^A(x)$ is the position vector in the five-dimensional Minkowskian embedding spacetime (with $A, B = 0, \ldots, 4$) corresponding to the point x of de Sitter. One can define the biscalar

$$Z(x,y) \equiv H^2 \eta_{AB}\xi^A(x)\xi^B(y) \tag{1}$$

$Z > 1$ for timelike separated points, $Z = 1$ for points connected by null geodesics, and $Z < 1$ for spacelike-separated points. It is worth emphasizing that there is no geodesic connecting two points with $Z < -1$. For points which are geodesically connected an alternative expression to (1) is [16]

$$Z(x, y) = \cos \sqrt{\frac{R\sigma(x, y)}{6}} \tag{2}$$

where $\sigma(x, y) \equiv \frac{1}{2}s^2(x, y)$, with $s(x, y)$ being the geodesic distance between the two points. We will use the closed coordinates $(\eta, \chi, \theta, \varphi)$, which cover the whole de Sitter spacetime. The line element reads

$$ds^2 = (H \sin \eta)^{-2} \left(-d\eta^2 + d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta \, d\varphi^2)\right) \tag{3}$$

where $\eta \in (0, \pi)$ and (χ, θ, φ) are the usual hyperspherical parametrizations of the S^3 spatial surfaces, which are invariant under the $O(4)$ subgroup of isometries.

A scalar field of mass m satisfies the Klein–Gordon equation $(\Box - m^2 - \xi R)\phi(x) = 0$, where ξ is a dimensionless constant which determines the coupling of the field to the spacetime curvature. We look for a set of mode solutions that can be written as $u_{klm}(x) = H \sin \eta \, X_k(\eta) Y_{klm}(\chi, \theta, \varphi)$, where Y_{klm} (with $k = 0, 1, 2, \ldots; l = 0, 1, \ldots, k - 1$; and $m = -l, \ldots, l$) are the S^3 spherical harmonics obeying $\Delta^{(3)} Y_{klm} = -k(k + 2)Y_{klm}$, which constitute a $(k + 1)^2$-dimensional representation of $O(4)$. Substituting into the Klein–Gordon equation, we find that $X_k(\eta)$ satisfy the equation

$$\frac{d^2 X_k}{d\eta^2} + \left\{(k + 1)^2 + (H \sin \eta)^{-2}\left[m^2 + \left(\xi - \frac{1}{6}\right)R\right]\right\}X_k = 0 \tag{4}$$

The general solution to this equation is

$$X_k(\eta) = (\sin \eta)^{-1/2}[A_k P^{\lambda}_{k+1/2}(-\cos \eta) + B_k Q^{\lambda}_{k+1/2}(-\cos \eta)] \tag{5}$$

where $\lambda \equiv \sqrt{(9/4) - (12/R)(m^2 + \xi R)}$, the coefficients A_k and B_k satisfy the normalization condition

$$A_k B_k^* - A_k^* B_k = i\,\frac{\Gamma\left(k+\frac{3}{2}-\lambda\right)}{\Gamma\left(k+\frac{3}{2}+\lambda\right)}$$

and $P^{\lambda}_{k+1/2}(z)$ and $Q^{\lambda}_{k+1/2}(z)$ are associated Legendre functions of the first and second kind, respectively. Given such a complete set of orthonormal solutions, one can expand the field operator as $\hat{\phi}(x) = \Sigma_{klm}\,(\hat{a}_{klm}u_{klm}(x) + \hat{a}^{+}_{klm}u^{*}_{klm}(x))$ and build the associated Fock space of states in the usual way. If we require the vacuum state to be $O(4, 1)$-invariant and of Hadamard type, A_k and B_k are uniquely determined [17, 18]:

$$A_k = \frac{\pi}{4}\,\frac{\Gamma\left(k+\frac{3}{2}-\lambda\right)}{\Gamma\left(k+\frac{3}{2}+\lambda\right)} \tag{6}$$

$$B_k = -\frac{2i}{\pi}\,A_k \tag{7}$$

The corresponding state is the so-called Euclidean vacuum, also known as the Bunch–Davies vacuum.

3. DISCUSSION OF THE CHOSEN STATE

The most natural state to choose is the Euclidean vacuum, which is invariant under the de Sitter group of isometries and is a Hadamard state. This is the one selected in quantum cosmology when the Hartle–Hawking no-boundary proposal or Vilenkin's tunneling wave function are taken as initial conditions [9]. In addition, when the field is massive, the field state asymptotically tends to it [8].

Both the Green's functions and the renormalized expectation value of the stress-energy tensor have been computed for the Euclidean vacuum in several works [19, 20]:

$$\begin{aligned} G^{(1)}(x,y) &\equiv \langle 0 | \{\hat{\phi}(x), \hat{\phi}(y)\} | 0 \rangle \\ &= \frac{2H^2}{(4\pi)^2}\,\Gamma\left(\frac{3}{2}-\lambda\right)\Gamma\left(\frac{3}{2}+\lambda\right)F\left(\frac{3}{2}-\lambda, \frac{3}{2}+\lambda, 2; \frac{1+Z(x,y)}{2}\right) \end{aligned} \tag{8}$$

$$\begin{aligned} \langle \hat{T}_{\mu\nu}(x)\rangle_{\rm ren} = -\frac{g_{\mu\nu}(x)}{64\pi^2}\Bigg\{ m^2 \left[m^2 + \left(\xi - \frac{1}{6}\right)R\right] \\ \times \left[\psi\left(\frac{3}{2}-\lambda\right) + \psi\left(\frac{3}{2}+\lambda\right) + \ln\frac{R}{12m^2}\right] \end{aligned}$$

$$- m^2 \left(\xi - \frac{1}{6}\right) R - \frac{1}{18} m^2 R - \frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 R^2 + \frac{R^2}{2160} \Bigg\} \tag{9}$$

where $F(a, b, c; z)$ is a hypergeometric function and $\psi(z) \equiv (d/dz) \ln \Gamma(z)$.

From (8) one can see that the symmetrized two-point function $G^{(1)}(x,y)$ has an infrared divergence (as $m^2 \to 0$) in the minimal coupling case. On the other hand, the expectation value of the stress-energy tensor remains finite, but is ambiguous, i.e., it depends on the way the limit $m^2 \to 0$, $\xi \to 0$ is taken. In fact, Allen [16] proved that in the massless, minimally coupled case there can be no Fock space built on a de Sitter-invariant vacuum. Allen and Folacci suggested that in this case one should consider instead $O(4)$-invariant states [17]. On the other hand, Kirsten and Garriga pointed out that such a peculiar behavior comes from the zero mode, which corresponds to a constant solution of the Klein–Gordon equation that appears when $k = 0$ in (4) [13]. They also showed that a special treatment of the zero mode seems to give an invariant vacuum. The Hilbert space of states is then the tensor product of the zero-mode part, which is equivalent to the Hilbert space for a one-dimensional nonrelativistic free particle times the Fock space corresponding to the nonzero modes (the whole space is not a Fock space, in agreement with Allen's result). The field operator is written

$$\hat{\phi}(x) = \sum_{\substack{klm \\ k \neq 0}} (\hat{a}_{klm} u_{klm}(x) + \hat{a}^{+}_{klm} u^{*}_{klm}(x))$$
$$+ \frac{H}{\sqrt{2}\pi} \left(\hat{Q}\frac{1}{2} + \hat{P}\left(\eta - \frac{1}{2}\sin 2\eta - \frac{\pi}{2}\right)\right) \tag{10}$$

where $\hat{Q}$ and $\hat{P}$ are the position and momentum operators of the free-particle Hilbert space and $\hat{a}^{+}_{klm}$ and $\hat{a}_{klm}$ are the creation and annihilation operators of the Fock space associated to all the modes with $k \neq 0$. Note that the two functions that multiply the operators $\hat{Q}$ and $\hat{P}$, when divided by $\sin\eta$, are solutions of Eq. (4) for $k = 0$ (with $m = \xi = 0$). In fact, the constant function multiplying the operator $\hat{Q}$ is precisely the aforesaid zero mode. These operators satisfy the usual commutation relations:

$$[\hat{a}_{klm}, \hat{a}_{k'l'm'}] = 0, \qquad [\hat{a}_{klm}, \hat{a}^{+}_{k'l'm'}] = \delta_{kk'}\delta_{ll'}\delta_{mm'}, \qquad [\hat{Q}, \hat{P}] = i \tag{11}$$
$$[\hat{a}_{klm}, \hat{Q}] = 0, \qquad [\hat{a}_{klm}, \hat{P}] = 0$$

The vacuum is defined as follows:

$$\hat{P}|0\rangle = 0 \tag{12}$$
$$\hat{a}_{klm}|0\rangle = 0, \qquad k \neq 0$$

As a consequence of the first condition, this vacuum is not normalizable, so strictly speaking it is not an element of the Hilbert space, but just a limit. However, as pointed out by Garriga and Kirsten, this fact is not so strange: it also happens for the ground state of a nonrelativistic free particle. These authors also computed the renormalized expectation value of the stress-energy tensor for this state:

$$\langle \hat{T}_{\mu\nu}(x)\rangle_{\rm ren} = \frac{119}{138240\pi^2} R^2 g_{\mu\nu}(x) = \frac{119}{960\pi^2} H^4 g_{\mu\nu}(x) \tag{13}$$

Two aspects should be stressed. First, the expression is de Sitter-invariant. Second, the energy density is lower than that of the $O(4)$-invariant states introduced by Allen and Folacci, the stress-energy tensor of which is not de Sitter-invariant. It is also remarkable that the expectation value for other states of observables such as the dispersion of the smeared field or the stress-energy tensor tend asymptotically ($\eta \to \pi$ or equivalently $t \to \infty$, where t is the proper time for a comoving observer) to the same value as that of the invariant vacuum [13].

4. COMPUTATION OF THE STRESS-ENERGY TENSOR FLUCTUATIONS

The quantity that characterizes the stress tensor fluctuations and which determines, via the Einstein–Langevin equation, the fluctuations in the space-time geometry [10] is the noise kernel, defined by the symmetrized two-point correlation function $\frac{1}{2}\langle\{\hat{t}_{\mu\nu}(x), \hat{t}_{\rho\sigma}(y)\}\rangle = \Re \langle \hat{t}_{\mu\nu}(x), \hat{t}_{\rho\sigma}(y)\rangle$, where $\hat{t}_{\mu\nu}(x) \equiv \hat{T}_{\mu\nu}(x) - \langle \hat{T}_{\mu\nu}(x)\rangle$ and $\hat{T}_{\mu\nu}(x) \equiv \lim_{x'\to x} \mathcal{D}_{\mu\nu}(x, x')(\hat{\phi}(x)\hat{\phi}(x'))$ with $\mathcal{D}_{\mu\nu}(x, x') \equiv (\nabla^x_\mu \nabla^{x'}_\nu - \frac{1}{2} g_{\mu\nu}(x)\nabla^x_\alpha\nabla^{x'\alpha})$ (we have chosen point-splitting regularization for convenience). It is easy to see that $\langle \hat{t}_{\mu\nu}(x)\hat{t}_{\rho\sigma}(y)\rangle$ is equivalent to $\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\rho\sigma}(y)\rangle - \langle \hat{T}_{\mu\nu}(x)\rangle\langle \hat{T}_{\rho\sigma}(y)\rangle$. Note that this expression is finite in the following sense: one can compute it suitably regularized; then the potentially divergent terms cancel and one can remove the regularization (letting $x' \to x$ and $y' \to y$):

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x)\hat{T}_{\rho\sigma}(y)\rangle &- \langle \hat{T}_{\mu\nu}(x)\rangle \langle \hat{T}_{\rho\sigma}(y)\rangle \\ &= \lim_{\substack{x'\to x\\ y'\to y}} \mathcal{D}_{\mu\nu}(x, x')\mathcal{D}_{\rho\sigma}(y, y') \Big(\langle \hat{\phi}(x)\hat{\phi}(x')\hat{\phi}(y)\hat{\phi}(y')\rangle \\ &\quad - \langle \hat{\phi}(x)\hat{\phi}(x')\rangle \langle \hat{\phi}(y)\hat{\phi}(y')\rangle\Big)\end{aligned} \tag{14}$$

The expression inside the parentheses is computed in the Appendix, where we find that it is finite, as pointed out above, and equals $G^+(x, y)G^+(x', y') + G^+(x, y')G^+(x', y)$. The final result after removing the regularization is

$$\langle \hat{t}_{\mu\nu}(x)\hat{t}_{\rho\sigma}(y)\rangle = [\nabla^y_\rho\nabla^x_\mu G^+(x, y)\nabla^y_\sigma\nabla^x_\nu G^+(x, y) + \nabla^y_\sigma\nabla^x_\mu G^+(x, y)\nabla^y_\rho\nabla^x_\nu G^+(x, y)$$
$$- g_{\mu\nu}(x)\nabla^y_\rho\, \nabla^x_\alpha G^+(x, y)\nabla^y_\sigma\, \nabla^{x\alpha} G^+(x, y)$$
$$- g_{\rho\sigma}(y)\nabla^y_\alpha\, \nabla^x_\mu G^+(x, y)\, \nabla^{y\alpha}\nabla^x_\nu G^+(x, y)$$
$$+ \frac{1}{2} g_{\mu\nu}(x)g_{\rho\sigma}(y)\, \nabla^y_\beta\nabla^x_\alpha G^+(x, y)\, \nabla^{y\beta}\nabla^{x\alpha} G^+(x, y)] \tag{15}$$

Thus, we need to determine the Wightman function $G^+(x, y) \equiv \langle 0|\hat{\phi}(x)\hat{\phi}(y)|0\rangle$ for our chosen state. If we consider the Euclidean vacuum for a massive, minimally coupled field and take the massless limit, the Wightman function diverges. As already explained, this is connected to the impossibility of having a Fock space built on a de Sitter-invariant vacuum in the massless case. To apply Garriga and Kirsten's special treatment of the zero mode, we have to consider separately the contribution from the zero mode and that from all the rest. To compute the latter, one can use the expression for the massive case, with the mass acting as a regulator of the infrared divergence, subtract the contribution from the zero mode, which contains all the infrared divergences, and then remove the regulator, i.e., take the massless limit.

When x and y are spacelike-separated, $G^+(x, y) = \frac{1}{2}(G^{(1)}(x, y) + G(x, y)) = \frac{1}{2}G^{(1)}(x, y) = \Re G^+(x, y)$ since the commutator $G(x, y) \equiv \langle 0|[\hat{\phi}(x), \hat{\phi}(y)]|0\rangle$ vanishes for causally disconnected points. In this case the noise kernel coincides with expression (15). We can also take advantage of the results in refs. 13 and 17 to write

$$G^{(1)}(x, y) = G^{(1)}_{\mathrm{NZM}}(x, y) + \frac{H^2}{8\pi^2}\langle 0|\hat{Q}^2|0\rangle \tag{16}$$

where

$$G^{(1)}_{\mathrm{NZM}}(x, y) \equiv \frac{R}{48\pi^2}\left[\frac{1}{1 - Z(x, y)} - \ln(1 - Z(x, y)) - \ln(4 \sin\eta_x \sin\eta_y) - \sin^2\eta_x - \sin^2\eta_y\right] \tag{17}$$

Several remarks are in order. $G^{(1)}_{\mathrm{NZM}}(x, y)$ corresponds to the nonzero-mode contribution, whereas the contribution from the zero mode reduces to $(H^2/8\pi^2)\langle 0|\hat{Q}^2|0\rangle$ since $\hat{P}|0\rangle = 0$. This term is actually divergent, but is independent of x and y, so that it will give no contribution to (15). Furthermore, the term $\ln(2\sin\eta_x) + \ln(2\sin\eta_y) + \sin^2\eta_x + \sin^2\eta_y$ will not contribute either, because in (15) there are always derivatives with respect to both x and y

acting on $G^{+}(x, y)$. Consequently, we only need to take into account the following part of the two-point function $G^{(1)}(x, y)$:

$$\mathcal{G}(x, y) \equiv \frac{R}{48\pi^2}\left[\frac{1}{1 - Z(x, y)} - \ln(1 - Z(x, y))\right] \tag{18}$$

If we consider spacelike-separated points which are geodesically connected, we can use expression (2) to derive the following results:

$$\nabla_{\mu}^{x}\sigma(x, y) = s(x, y)s_{\mu}(x) \tag{19}$$

$$\nabla_{\rho}^{y}\nabla_{\mu}^{x}\sigma(x, y) = s_{\rho}(y)s_{\mu}(x) \tag{20}$$

where $s^{\mu}(x)$ is the unit vector tangent at point x to the geodesic joining x and y. The derivatives of $Z(x, y)$ are then

$$\nabla_{\mu}^{x}Z(x, y) = \sqrt{\frac{R}{12}}\,(1 - Z^2(x, y))\, s_{\mu}(x) \tag{21}$$

$$\nabla_{\rho}^{y}\nabla_{\mu}^{x}Z(x, y) = \frac{R}{12}(1 - Z^2(x, y))\, s_{\rho}(y)s_{\mu}(x) \tag{22}$$

Thus, after a few algebraic manipulations, we have

$$\nabla_{\rho}^{y}\nabla_{\mu}^{x}\, G^{+}(x, y) = \nabla_{\rho}^{y}\nabla_{\mu}^{x}\mathcal{G}(x, y) = \frac{H^4}{4\pi^2}\,\frac{1 + 4Z(x, y)}{(1 - Z(x, y))^2}\, s_{\rho}(y)s_{\mu}(x) \tag{23}$$

Substituting this result into (15), we get our final result for the noise kernel corresponding to spacelike-separated points:

$$\begin{aligned}\Re\langle \hat{t}_{\mu\nu}(x)\, \hat{t}_{\rho\sigma}(y)\rangle = {} & \frac{H^8}{16\pi^4}\left(\frac{1 + 4Z(x, y)}{(1 - Z(x, y))^2}\right)^2 \\ & \times [2s_{\mu}(x)s_{\nu}(x)\, s_{\rho}(y)s_{\sigma}(y) + g_{\mu\nu}(x)\, s_{\rho}(y)s_{\sigma}(y) \\ & + g_{\rho\sigma}(y)\, s_{\mu}(x)s_{\nu}(x) + \frac{1}{2}\, g_{\mu\nu}(x)\, g_{\rho\sigma}(y)]\end{aligned} \tag{24}$$

5. CONCLUSION

Comparing (24) with the "square" of (13), one realizes that the contribution from the stress-energy fluctuations is at least as important as that coming from the expectation value. Note that in order to compare both expressions it is convenient to consider the metric and the tangent vector components as referred to an orthonormal base. Of course, given that $H^4 << m_p^2H^2$, this stochastic backreaction source is still much smaller than the dominant term which drives inflation (usually coming from the potential of the inflaton scalar field) and which is responsible for the near de Sitter geometry of the spacetime. It seems interesting to point out that the expectation value (13) yields a negative value for the energy density because of two facts. First, Ford and collaborators (see, for instance, ref. 6) have suggested that one may expect important stress-energy fluctuations especially in those cases where the energy density is negative. Second, this is in agreement with some of the results found in ref. 12.

It is only the contribution to the variations of the potential coming from fluctuations of the inflaton field which are usually considered when addressing the generation of large-scale gravitational inhomogeneities. Expressions of the sort $\langle \phi^2(x) \rangle = (H^3/4\pi^2)\Delta t$ or related ones [21], where proper infrared (related to initial conditions) and ultraviolet cutoffs (only scales larger than the horizon are considered: $\lambda_{phys} > H^{-1}$) have been imposed, are of frequent use in the literature [1, 22]. These fluctuations for the smeared inflaton field can be interpreted as if the smeared field were undergoing a sort of "Brownian motion" [24]. Such an interpretation still applies to Garriga and Kirsten's vacuum, as they show when analyzing the dispersion of the smeared field [13]. The fluctuations associated with a given cosmic scale arise from the stochastic contribution, when leaving the horizon, from the modes of the inflaton field perturbations with a wavelength corresponding to that scale. The usual treatment just deals with first-order variations of the potential, which are assumed to provide the dominant contribution, and does not take into account the backreaction contribution from the kinetic terms of the quantum perturbations of the inflaton field [1]. The latter can be modeled, at least partially, by a free, massless, minimally coupled quantum scalar field. This is precisely the case we considered in this paper. Furthermore, as mentioned above, we have found that the contribution from the fluctuations of the stress-energy tensor can be as important as the expectation value itself. Within the inflationary context, this contribution to backreaction becomes especially relevant above the "self-reproduction" scale and could have important implications for the stochastic inflation approach, where this effect is never taken into account.

One can try to extract information about large-scale fluctuations from correlations for spacelike-separated points which are beyond the horizon distance [23]. Unfortunately, for such points $Z(x, y) < -1$ and there is no geodesic connecting them, and thus one cannot directly use our results, which were obtained using (19) and (20), which in turn rely on the particular expression (2) for geodesically connected points. If one is interested in the $Z(x, y) < -1$ case, then the general expression (1) should be used. The relative importance of the fluctuations that we have found for geodesically connected points encourages us to pursue this research further.

APPENDIX

In this appendix we compute $\langle\hat{\phi}(x)\hat{\phi}(x')\hat{\phi}(y)\hat{\phi}(y')\rangle - \langle\hat{\phi}(x)\hat{\phi}(x')\rangle\langle\hat{\phi}(y)\hat{\phi}(y')\rangle$ in the context of Garriga and Kirsten's treatment. So we will use expression (10) for the field $\hat{\phi}(x)$ and take the expectation values with respect to the vacuum state defined in (12). First, we simply rewrite (10) for convenience:

$$\hat{\phi}(x) = \sum_{\substack{klm \\ k\neq 0}} (\hat{a}_{klm}\, u_{klm}(x) + \hat{a}^{+}_{klm} u^{*}_{klm}(x)) + \left(\hat{Q}\,\frac{u_{000}(x)}{\langle 0|\hat{Q}^2|0\rangle^{1/2}} + \hat{P}\, v_0(x)\right) \tag{A1}$$

Using the commutation relations (11) and (12), we find

$$\langle\hat{\phi}(x)\hat{\phi}(x')\hat{\phi}(y)\,\hat{\phi}(y')\rangle = \sum_{klm}\sum_{k'l'm'} (u_{klm}(x)\, u^{*}_{klm}(y)\, u_{k'l'm'}(x')\, u^{*}_{k'l'm'}(y') + u_{klm}(x)\, u^{*}_{klm}(y')\, u_{k'l'm'}(x')\, u^{*}_{k'l'm'}(y) + \sum_{klm} u_{klm}(x)\, u^{*}_{klm}(x') \sum_{k'l'm'} u_{k'l'm'}(y)\, u^{*}_{k'l'm'}(y')$$

and

$$\langle\hat{\phi}(x)\hat{\phi}(x')\rangle\,\langle\hat{\phi}(y)\hat{\phi}(y')\rangle = \sum_{klm} u_{klm}(x)\, u^{*}_{klm}(x') \sum_{k'l'm'} u_{k'l'm'}(y)\, u^{*}_{k'l'm'}(y') \tag{A4}$$

From this we obtain

$$\langle\hat{\phi}(x)\,\hat{\phi}(x')\hat{\phi}(y)\hat{\phi}(y')\rangle - \langle\hat{\phi}(x)\hat{\phi}(x')\rangle\,\langle\hat{\phi}(y)\,\hat{\phi}(y')\rangle = G^{+}(x, y)G^{+}(x', y') + G^{+}(x, y')\, G^{+}(x', y) \tag{A5}$$

where $G^{+}(x, y)$ is the Wightman two-point function

$$G^{+}(x, y) = \sum_{\substack{klm \\ k\neq 0}} u_{klm}(x)\, u^{*}_{klm}(y) + u_{000}(x)\, u^{*}_{000}(y)\,\langle 0|\hat{Q}^2|0\rangle \tag{A6}$$

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